UCL 2024/25: Monte Carlo Simulation of the Knockout Stage

Ethan Robertson

Abstract

Building upon my previous analysis of team strength using an Elo rating system, I developed a Monte Carlo simulation to forecast outcomes for the knockout stage of the 2024/25 UEFA Champions League (UCL). This simulation leverages probability estimates derived from Elo ratings to model the tournament's progression under a range of possible scenarios. The primary objective of this study is to estimate each team's likelihood of advancing through the knockout rounds and ultimately winning the competition. Additionally, I develop an optimal betting strategy using the explicit solution to the Kelly Criterion as formulated by Smoczynski & Tomkins, allowing for the optimal allocation of a fixed bankroll across multiple bets while maximizing expected logarithmic growth of wealth.

1 Introduction

Building upon my previous analysis of team strength using an Elo rating system, I developed a Monte Carlo simulation to forecast outcomes for the knockout stage of the 2024/25 UEFA Champions League (UCL). This simulation leverages probability estimates derived from Elo ratings to model the tournament's progression under a range of possible scenarios.

The primary objective of this study is to estimate each team's likelihood of advancing through the knockout rounds and ultimately winning the competition. By simulating the tournament structure repeatedly, I obtain a probabilistic distribution of outcomes, allowing for a quantitative assessment of each team's winning chances.

Additionally, I aim to develop an optimal betting strategy using the explicit solution to the Kelly Criterion as formulated by Smoczynski & Tomkins.[1] This approach allows for the optimal allocation of a fixed bankroll across multiple bets, maximizing the expected

[1]Smoczynski, K., & Tomkins, J. (2004). An explicit solution to the multivariate Kelly problem. *Stochastic Processes and Their Applications*, 112(2), 197–213.

logarithmic growth of wealth.

For details on the Elo rating system and its application, see *UCL Elo Ratings & Predictive Modeling (2024/25 Season)*.

2 Methodology

2.1 Elo-Based Probability Model

The foundation of the simulation is an Elo rating system assigning each team a numerical score. At the end of the league phase, updated Elo ratings were used to estimate head-to-head probabilities. The probability of Team A winning against Team B is calculated using the following formula:

$$P_A = \frac{1}{1 + 10^{(R_B - R_A)/400}} \tag{1}$$

In this equation, P_A represents the probability of Team A winning, while R_A and R_B denote the Elo ratings for Teams A and B, respectively. The scaling factor of 400 controls the sensitivity to rating differences, ensuring reasonable probability distributions across varying rating gaps.

2.2 Monte Carlo Simulation Framework

The simulation process involved running 1,000,000 Monte Carlo simulations of the knockout stage. For each match simulation, a random number u was drawn from a uniform distribution $\mathcal{U}(0, 1)$. If $u < P_A$, Team A was deemed to advance; otherwise, Team B progressed. This process was repeated for each match in the tournament, from the Round of 16 through to the Final. The advancement rates for each team were carefully recorded and aggregated to produce the final probability estimates.

3 Results and Key Insights

3.1 Probability of Advancing by Stage

3.2 Value Betting Analysis

Through a comparative analysis of model-derived probabilities and implied bookmaker probabilities, several notable value gaps were identified. Figure 2 illustrates these differences for selected teams.

Team	QF Probability	SF Probability	Final Probability	Win Probability
FC Barcelona	62.15%	34.10%	19.49%	10.58%
FC Internazionale Milano	62.58%	34.69%	17.95%	9.24%
Arsenal FC	54.14%	27.53%	15.47%	8.44%
Club Atlético de Madrid	51.69%	26.83%	15.12%	8.26%
Borussia Dortmund	52.09%	26.45%	14.13%	7.12%
Paris Saint-Germain FC	51.44%	28.67%	13.75%	7.03%
Real Madrid CF	48.31%	24.19%	13.20%	7.00%
Liverpool FC	48.56%	26.43%	12.28%	6.13%
Bayer 04 Leverkusen	52.89%	26.78%	12.78%	6.06%
Lille OSC	47.91%	23.30%	11.97%	5.78%
PSV	45.86%	21.45%	11.21%	5.67%
FC Bayern München	47.11%	22.52%	10.10%	4.53%
Club Brugge KV	50.93%	23.09%	9.85%	4.51%
Sport Lisboa e Benfica	49.07%	21.81%	9.11%	4.11%
Aston Villa FC	37.85%	16.16%	7.28%	3.07%
Feyenoord Rotterdam	37.42%	16.00%	6.28%	2.46%

Figure 1: Simulated probabilities for each team advancing to the quarterfinals, semifinals, final, and winning the UCL.

Team	Implied Probability	Elo Probability	Edge
Borussia Dortmund	1.96%	7.12%	5.16%
PSV	0.66%	5.67%	5.01%
Lille OSC	0.99%	5.78%	4.79%
Club Brugge KV	0.66%	4.51%	3.85%
Club Atlético de Madrid	4.76%	8.26%	3.50%
Bayer 04 Leverkusen	2.94%	6.06%	3.12%
Sport Lisboa e Benfica	1.49%	4.11%	2.62%
Feyenoord Rotterdam	0.50%	2.46%	1.96%
FC Internazionale Milano	7.69%	9.24%	1.55%
Aston Villa FC	2.94%	3.07%	0.13%
Paris Saint-Germain FC	7.69%	7.03%	-0.66%
Arsenal FC	14.29%	8.44%	-5.85%
FC Bayern München	12.50%	4.53%	-7.97%
Liverpool FC	15.38%	6.13%	-9.25%
FC Barcelona	20.00%	10.58%	-9.42%
Real Madrid CF	23.09%	7.00%	-16.09%

Figure 2: Differences between model-implied probabilities and bookmaker-implied probabilities for selected teams.

The analysis revealed FC Barcelona as the tournament favorite, with a 10.58% probability of winning the competition according to our simulations. Additionally, significant value opportunities were identified in the odds for Dortmund, PSV, and Lille OSC when compared against VegasInsider odds.

3.3 Kelly Criterion-Based Bankroll Management

To translate model-derived probabilities into actionable betting decisions, I employed the Kelly Criterion, a mathematical approach to bankroll optimization. Specifically, I used the explicit solution outlined by Smoczynski & Tomkins, which extends the classic Kelly framework to multiple simultaneous bets with correlated outcomes—a more realistic scenario in tournament-based sports betting.

3.3.1 Theoretical Foundation

The Kelly Criterion seeks to maximize the expected logarithmic growth of wealth. For a single bet, the optimal fraction f^* of the bankroll to wager is determined by:

$$f^* = bp - q \tag{2}$$

where f^* represents the fraction of bankroll to wager, b denotes the decimal odds minus 1 (net odds), p is the probability of winning derived from the model, and $q = 1 - p$ represents the probability of losing.

In the context of multiple concurrent bets, as is common in tournament settings, the optimization problem becomes more complex. Smoczynski & Tomkins derive a closed-form solution as a constrained optimization problem that maximizes expected log wealth subject to the constraint that the sum of all bet fractions does not exceed the total bankroll.

3.3.2 Implementation Procedure

The implementation process began with the identification of value bets by comparing the model-derived probability of winning the UCL (p_i) with the implied probability based on bookmaker odds ($\hat{p}_i = \frac{1}{\text{odds}_i}$) for each team i. A bet was considered to have positive expected value when $p_i > \hat{p}_i$.

The optimal fractions $\{f_i^*\}$ were determined by solving the optimization problem:

$$\max_f \ \mathbb{E}\left[\log\left(1 + \sum_i f_i X_i\right)\right] \tag{3}$$

subject to:

$$\sum_i f_i \leq 1, \quad f_i \geq 0 \quad \forall i \tag{4}$$

where X_i represents the net payoff from bet i, taking the value b_i if bet i wins and -1 otherwise. The optimization was implemented using numerical methods in Python.

Table 1: Optimal Betting Fractions Based on Kelly Criterion and Model Probabilities

Team	**Implied Prob.**	**Elo Prob.**	**Eri**	**Optimal Bet**
PSV	0.66%	5.67%	9.07	12.46%
Club Brugge KV	0.66%	4.51%	7.01	12.74%
Lille OSC	0.99%	5.78%	5.86	12.43%
Feyenoord Rotterdam	0.50%	2.46%	4.81	13.25%
Borussia Dortmund	1.96%	7.12%	3.27	12.10%
Sport Lisboa e Benfica	1.49%	4.11%	2.24	12.84%
Bayer 04 Leverkusen	2.94%	6.06%	1.42	12.36%
Club Atlético de Madrid	4.76%	8.26%	1.04	11.82%

3.4 Robustness Testing

To assess the model's sensitivity to potential rating biases, I conducted a series of Monte Carlo simulations under various scenarios:

- Base case with no rating adjustments
- Premier League teams overrated by 40 Elo points
- La Liga teams underrated by 30 Elo points
- Bundesliga teams overrated by 50 Elo points

The results revealed remarkable consistency across scenarios, with ROIs ranging from 223% to 280%. The Premier League overrated scenario performed best (279.68% ROI), suggesting potential conservative bias in our initial Premier League team assessments. The La Liga underrated scenario showed the lowest but still substantial returns (223.07% ROI), indicating robustness against potential underestimation of Spanish teams.

3.5 Risk Management Considerations

While the Kelly Criterion provides theoretically optimal bet sizing, its aggressive nature often necessitates practical adjustments. A more conservative approach using fractional Kelly betting is recommended, where positions are scaled to a fixed proportion (e.g., 50%)

of the optimal amounts. This modification helps mitigate volatility while still maintaining significant expected returns over the long term.

The robustness testing results suggest that the strategy maintains profitability even under significant rating biases. However, to further enhance risk management, I propose a diversified approach:

- 50% of bankroll allocated to traditional win bets on undervalued teams
- 40% allocated to synthetic positions (laying overvalued teams)
- 10% reserved for dynamic hedging opportunities

This allocation strategy aims to capture value from both sides of the market while maintaining a buffer for risk management. The synthetic positions are particularly valuable in cases where teams are significantly overvalued by the market, as they allow us to profit from both overpricing and underpricing scenarios.

3.6 Distribution Analysis and Risk Considerations

A critical examination of the payout distribution reveals a fundamental challenge in the original betting strategy. As shown in Figure 3, the distribution exhibits significant right-skew, indicating that while the expected value remains positive, it is driven primarily by a small number of extremely high payouts occurring with low probability. This characteristic, while mathematically valid, presents practical challenges for risk management and capital allocation.

The Value at Risk (VaR) analysis at the 95% confidence level quantifies this risk, showing that the strategy could experience substantial drawdowns in the majority of scenarios. This insight prompted the development of a more balanced approach that considers both expected value and the probability of success, while maintaining the ability to exploit market mispricing.

3.7 Refined Betting Strategy

To address these challenges while preserving the strategy's core advantages, I developed a three-component approach that balances risk and return:

3.7.1 Traditional Win Bets (50% of bankroll)

This component focuses on teams that satisfy two key criteria:

- Positive expected value based on model-derived probabilities
- Minimum probability threshold (e.g., 5% chance of winning)

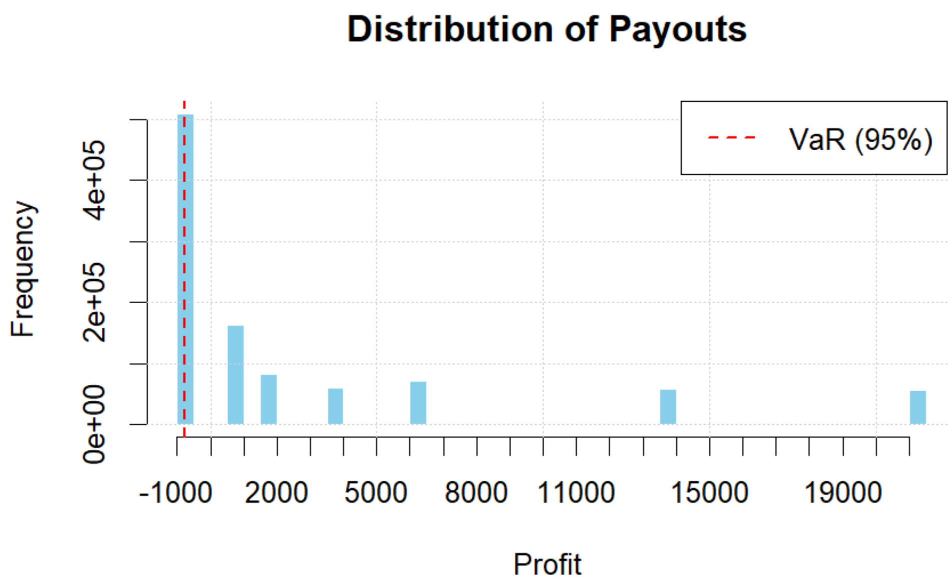

Figure 3: Distribution of betting payouts showing right-skewed nature of returns. The red line indicates the Value at Risk (VaR) at 95% confidence level.

By implementing these filters, we ensure a more balanced distribution of potential outcomes while maintaining exposure to undervalued teams. The probability threshold helps avoid excessive reliance on low-probability, high-payout scenarios.

3.7.2 Synthetic Positions (40% of bankroll)

The synthetic component is implemented through two primary mechanisms:

1. **Lay Bets**: Directly laying overvalued teams on betting exchanges, particularly those with implied probabilities significantly higher than model estimates. This allows us to profit from both overpricing and underpricing scenarios.
2. **Combination Bets**: Creating synthetic positions through combinations of traditional bets. For example, betting against a team can be achieved by:
   - Backing all other teams in the tournament
   - Using accumulator bets with carefully selected combinations
   - Implementing spread betting strategies where available

The synthetic positions are particularly valuable in cases where teams are significantly overvalued by the market, as they allow us to profit from both overpricing and underpricing scenarios.

3.7.3 Synthetic Lay Betting Implementation

Building upon the synthetic positions framework, we implemented a specific strategy focusing on teams that are overvalued by the market. The strategy identifies teams where the market-implied probability exceeds our model probability by at least 5 percentage points, creating synthetic lay positions against these teams.

For the 2024/25 UCL, our analysis identified five significantly overvalued teams:

Table 2: Overvalued Teams and Synthetic Lay Positions

Team	**Model Prob.**	**Market Prob.**	**Overvaluation**	**Lay Stake**
Real Madrid CF	12.00%	23.09%	+11.09%	$100
FC Barcelona	10.58%	20.00%	+9.42%	$100
Liverpool FC	7.50%	15.38%	+7.88%	$100
Arsenal FC	8.00%	14.29%	+6.29%	$100
FC Bayern München	6.50%	12.50%	+6.00%	$100

The strategy creates synthetic lay positions against these teams, with each position sized at $100. Given the mutually exclusive nature of tournament outcomes (only one team can win), the maximum potential loss is limited to the largest single liability ($700 in this case), rather than the sum of all liabilities. This creates a favorable risk-reward profile:

- Total Stake: $500 (5 positions × $100)
- Maximum Liability: $700 (if Bayern wins)
- Best Case: Win $500 (if none of these teams win)
- Probability of No Winner: 55.92% (1 - 44.08%)

This approach provides a balanced risk-reward profile while maintaining the ability to profit from market mispricing. The strategy is particularly effective because it:

- Focuses on significant probability discrepancies ($\geq 5\%$)
- Limits exposure through position sizing
- Benefits from the mutually exclusive nature of tournament outcomes
- Maintains a favorable probability of success (55.92%)

3.7.4 Dynamic Hedging (10% of bankroll)

The final component provides flexibility to manage risk exposure throughout the tournament:

- Real-time position adjustment based on match outcomes

- Opportunistic hedging when favorable odds present themselves
- Contingency planning for various tournament scenarios

This refined approach maintains the mathematical rigor of the original strategy while addressing its practical limitations. By incorporating synthetic positions, we can capture value from overvalued teams without relying solely on low-probability, high-payout scenarios. The dynamic hedging component further enhances risk management capabilities.

4 Conclusion

The combination of Monte Carlo simulation and Kelly optimization framework provides a robust methodology for forecasting UCL knockout outcomes and developing profitable betting strategies. The analysis reveals significant opportunities for value betting, particularly in cases where market odds diverge substantially from model-derived probabilities. However, the implementation of such strategies requires careful consideration of risk management principles, given the inherent uncertainties in probabilistic modeling and the potential for substantial variance in outcomes.